EE 232: Lightwave Devices

Lecture #22 – Photodetector noise

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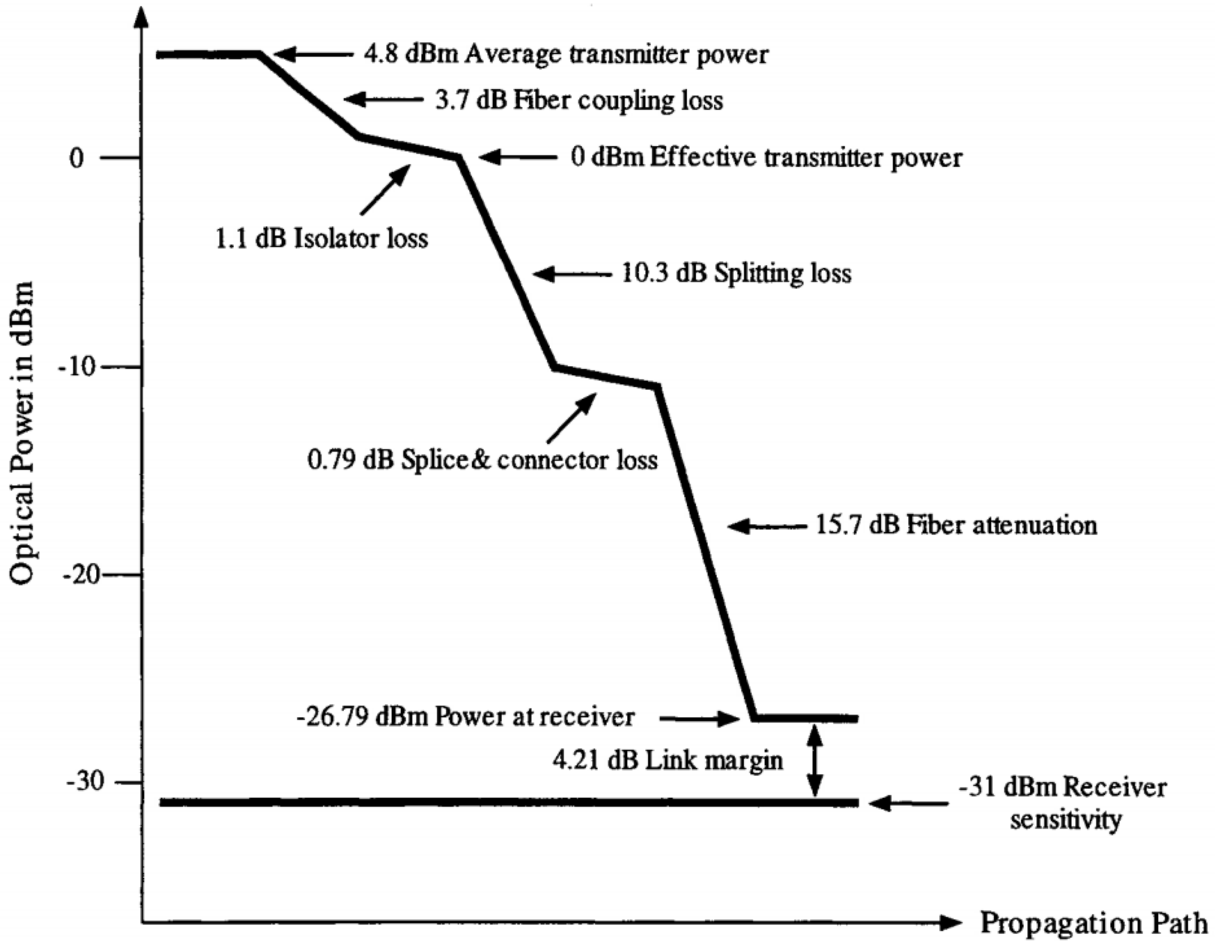
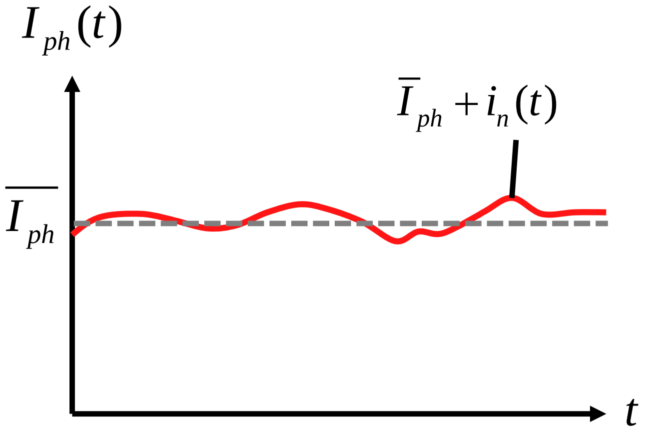
Optical link budget example



Sensitivity also specified in photons per bit: $N_b = \frac{P_{opt}}{h\nu} \frac{1}{\text{data rate}}$

Noise



photocurrent $$I_{ph}(t) = \underbrace{\bar{I}_{ph}}_{\text{average photocurrent}} + \underbrace{i_n(t)}_{\text{noise}}$$

Noise power dissipated into resistive load

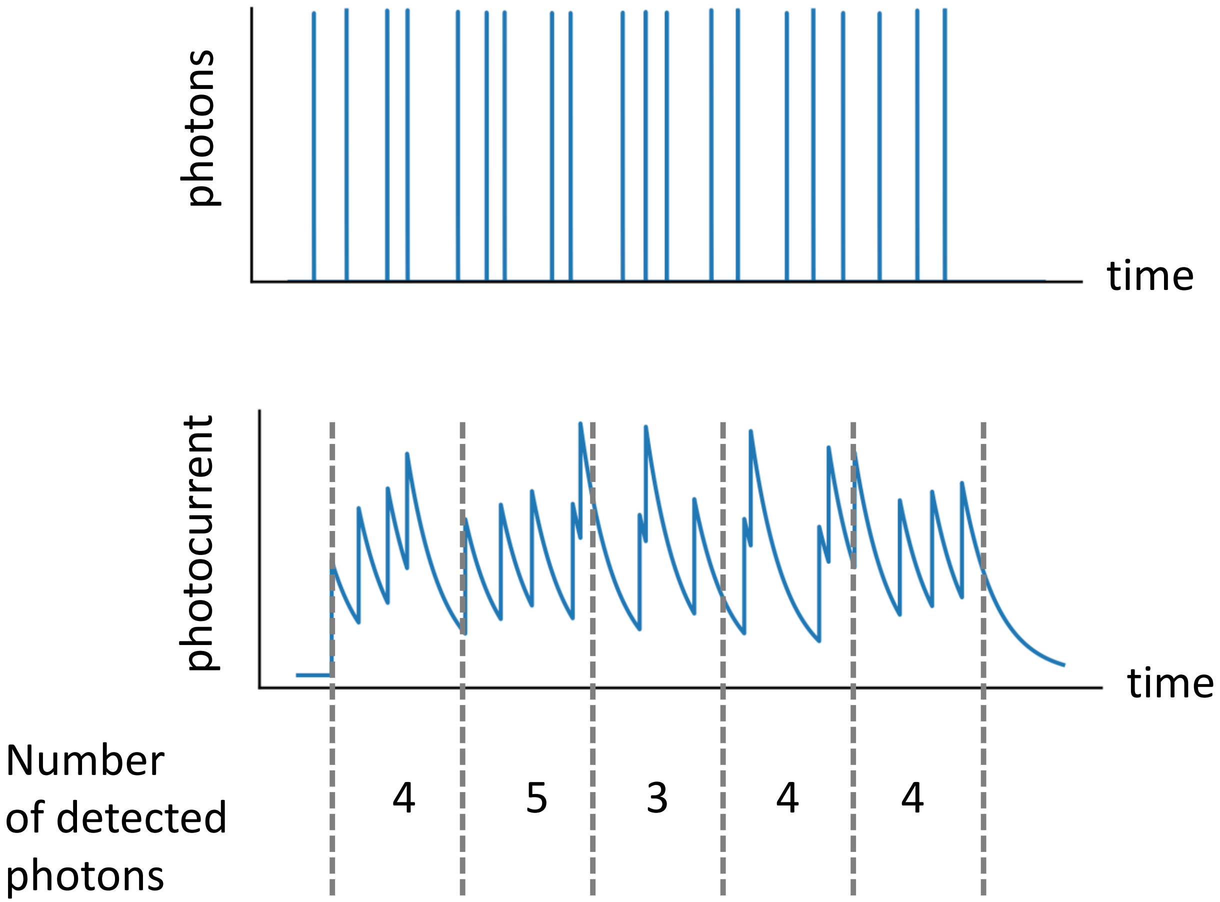
$$p_{avg} = \overline{i_n^2(t)R}$$ (bar denotes average)

$$= \underbrace{\overline{i_n^2(t)}}_{\text{noise power}}R$$

$$\overline{i_n^2(t)} = \frac{1}{T}\int_0^T i_n^2(t)\,dt$$ (RMS squared value of the noise)

Shot noise

Most fundamental noise source. Sets sensitivity limit for conventional optical receivers. Consequence of particle-like nature of photon.

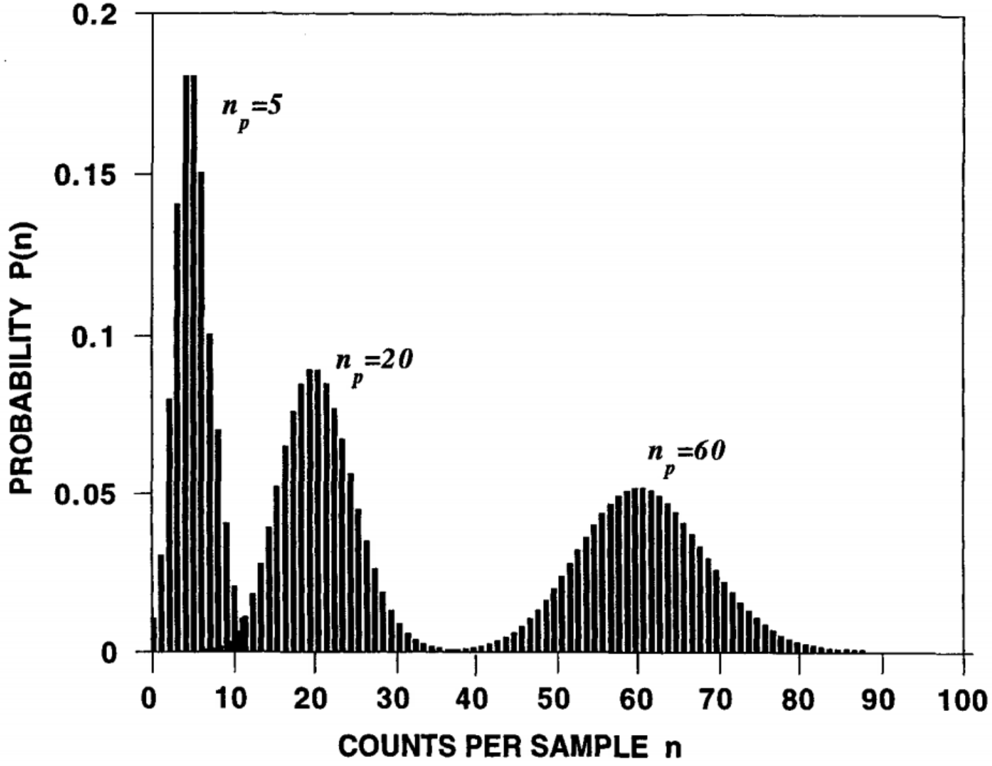


Poisson statistics

The probability of detecting *N* photons is governed by the Poisson distribution

$$P(N) = \frac{\bar{N}^{N}}{N!} e^{-\bar{N}}$$

$\bar{N}$: average number of photons during observation period

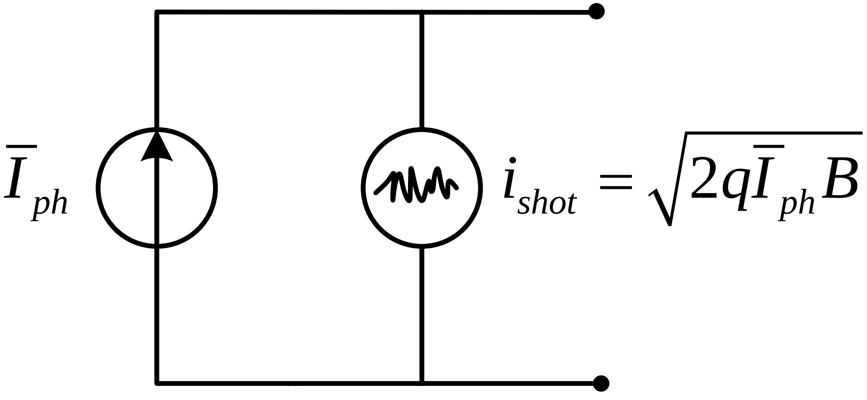


Shot noise power

$$i_{shot}^2 = \overline{i_{shot}^2(t)} = 2q\overline{I}_{ph}B \quad \text{(amps}^2\text{)}$$

average value of photocurrent

bandwidth of measurement



$$i_{shot} = \sqrt{2q\overline{I}_{ph}B}$$

Photodetector equivalent circuit

Shot noise power **increases** with higher average photocurrent.
This is a consequence of Poisson statistics.

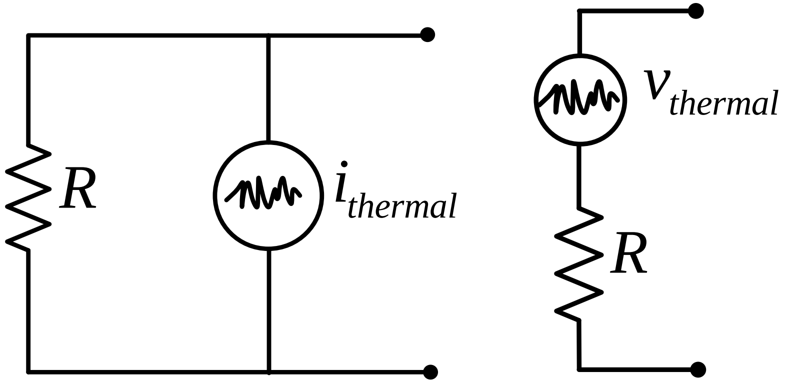
The observation time is reduced as the bandwidth increases. This increases the likelihood that the photocurrent measured within the observation time is different than the average photocurrent and thus increases the noise power.

Thermal noise

All physical resistances have fluctuating voltage as a result of thermal motion of charged carriers.

$$v_{thermal}^2(t) = 4kTBR$$

$$i_{thermal}^2 = 4kTBR^{-1}$$

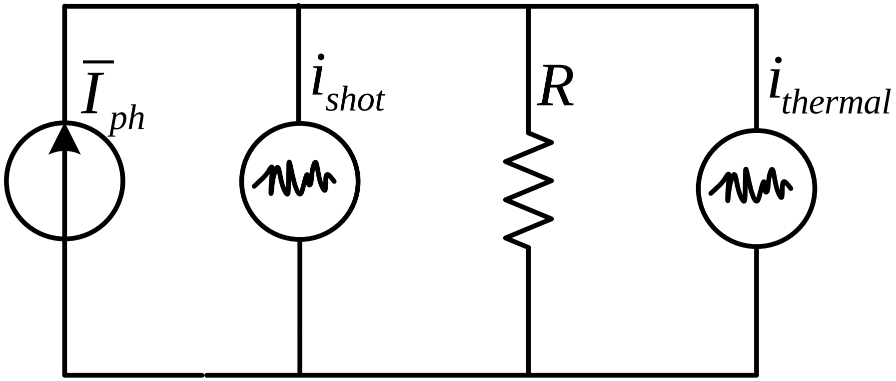


Resistance equivalent circuit

The photodetector may have resistance which can contribute to thermal noise (series resistance or junction shunt resistance) but often the largest contribution to thermal noise comes from the amplifier connected to the photodetector.

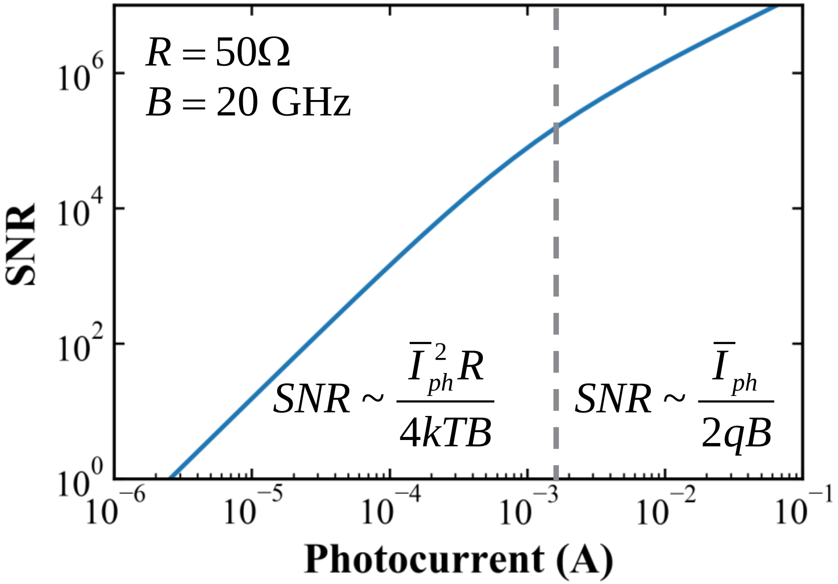
p-i-n photodiode noise

$$SNR = \frac{\bar{I}_{ph}^2}{i_{shot}^2 + i_{thermal}^2} = \frac{\bar{I}_{ph}^2}{2q\bar{I}_{ph}B + 4kTBR^{-1}} = \boxed{\frac{\left(\frac{\eta q}{h\nu}P_{opt}\right)^2}{2q\frac{\eta q}{h\nu}P_{opt}B + 4kTBR^{-1}}}$$



Simplified equivalent circuit
of photoreceiver with p-i-n photodiode

Resistance includes junction resistance and resistance of amplifier stage



Excess noise in APDs

Multiplication factor (M) is a random variable and can fluctuate about some average value. The shot noise power for an APD can be written

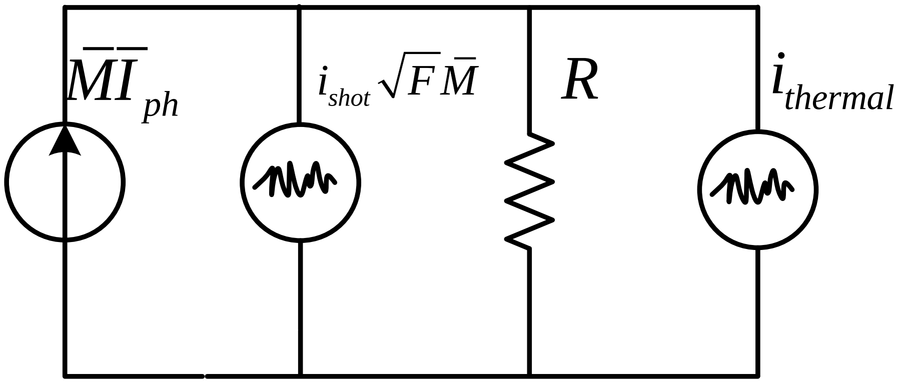
$$i^2_{shot,APD} = i^2_{shot}\bar{M}^2 F = 2q\bar{I}_{ph}\bar{M}^2 FB$$

Average multiplication factor

Excess noise factor

$$F = \frac{\overline{M^2}}{\bar{M}} = k\bar{M}_n + (1-k)\left(2 - \frac{1}{\bar{M}_n}\right)$$

We desire k to be small for small excess noise factor



Simplified equivalent circuit of photoreceiver with APD

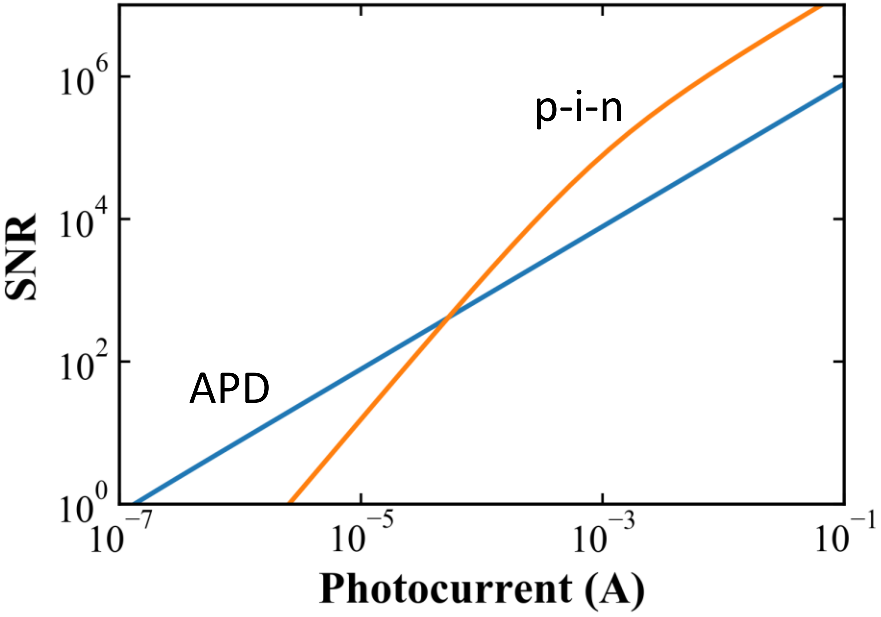
Comparing p-i-n and APD noise

$$SNR_{APD} = \frac{\bar{M}^2 \bar{I}_{ph}^2}{i_{shot}^2 \bar{M}^2 F + i_{thermal}^2} = \frac{\bar{M}^2 \bar{I}_{ph}^2}{2q\bar{I}_{ph}\bar{M}^2 FB + 4kTBR^{-1}}$$

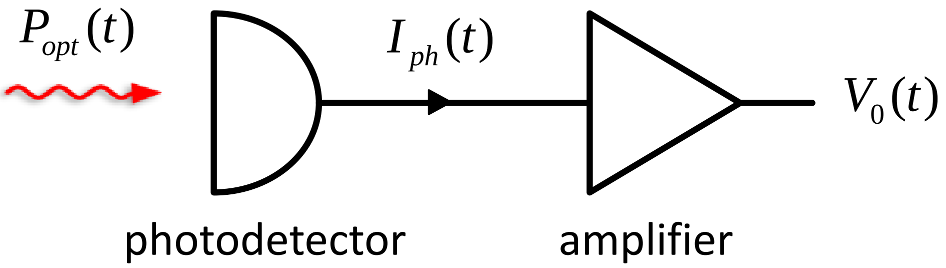
$$SNR_{p-i-n} = \frac{\bar{I}_{ph}^2}{i_{shot}^2 + i_{thermal}^2} = \frac{\bar{I}_{ph}^2}{2q\bar{I}_{ph}B + 4kTBR^{-1}}$$

$$\boxed{\frac{SNR_{APD}}{SNR_{p-i-n}} \cong \frac{1}{F}\left(1 + \frac{i_{thermal}^2}{i_{shot}^2}\right)}$$

APD has better SNR when thermal noise dominates



Direct detection



Photocurrent can be written as

$$I_{ph}(t) = \frac{\eta q}{h\nu} P_{opt}(t) = \frac{\eta q}{h\nu} \frac{1}{Z_0} \left| E_{opt}(t) \right|^2$$

$$= \frac{\eta q}{h\nu} \frac{1}{Z_0} E_{opt}^2 \cos^2(\omega_{opt} t + \phi)$$

Define average power as $P_{opt} = \overline{P_{opt}(t)} = \frac{1}{2Z_0} E_{opt}^2$

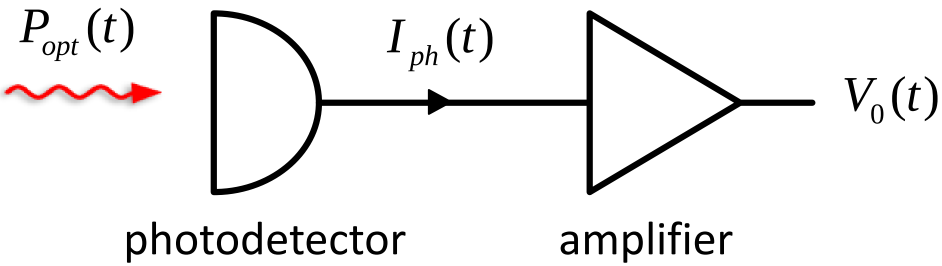
Then, $I_{ph}(t) = \frac{\eta q}{h\nu} 2P_{opt} \cos^2(\omega_{opt} t + \phi)$

$$= \frac{\eta q}{h\nu} P_{opt} [1 + \cos 2(\omega_{opt} t + \phi)]$$

The photodetector is too slow to respond to the time-varying field →

$$\boxed{I_{ph}(t) = \frac{\eta q}{h\nu} P_{opt}}$$

Direct detection



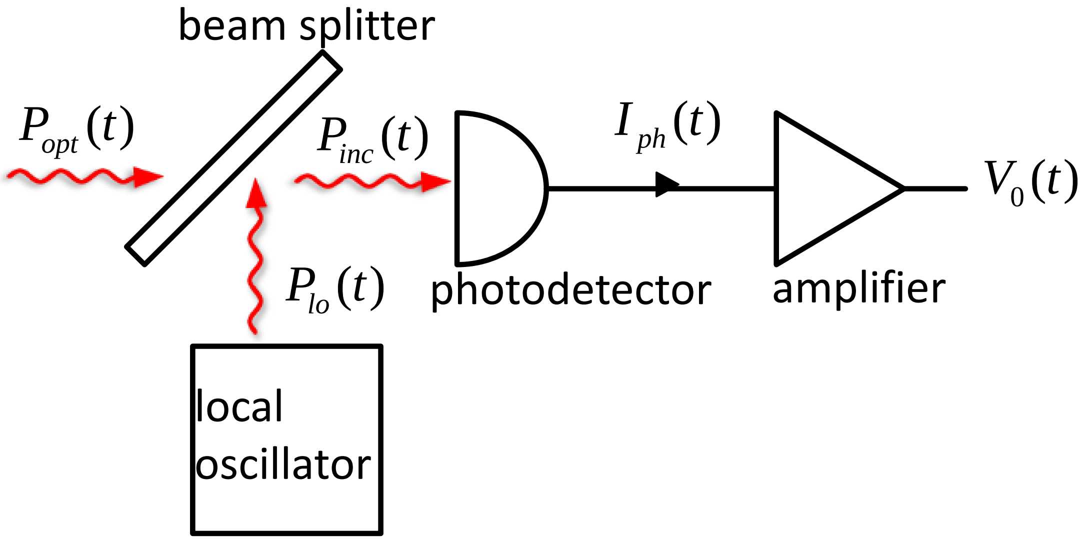
$$I_{ph}(t) = \frac{\eta q}{h\nu} P_{opt}$$

Direct detection receivers respond only to changes in the intensity of the incident field.

$$SNR_{dd} = \frac{I_{ph}^2}{i_{shot}^2 + i_{thermal}^2} = \frac{I_{ph}^2}{2qI_{ph}B + 4kTBR^{-1}} = \boxed{\frac{\left(\frac{\eta q}{h\nu} P_{opt}\right)^2}{2q\frac{\eta q}{h\nu} P_{opt} B + 4kTBR^{-1}}}$$

(assuming p-i-n photodiode)

Coherent detection



Photocurrent can be written as

$$I_{ph}(t) = \frac{\eta q}{h\nu}\frac{1}{Z_0}\left|E_{inc}(t)\right|^2$$

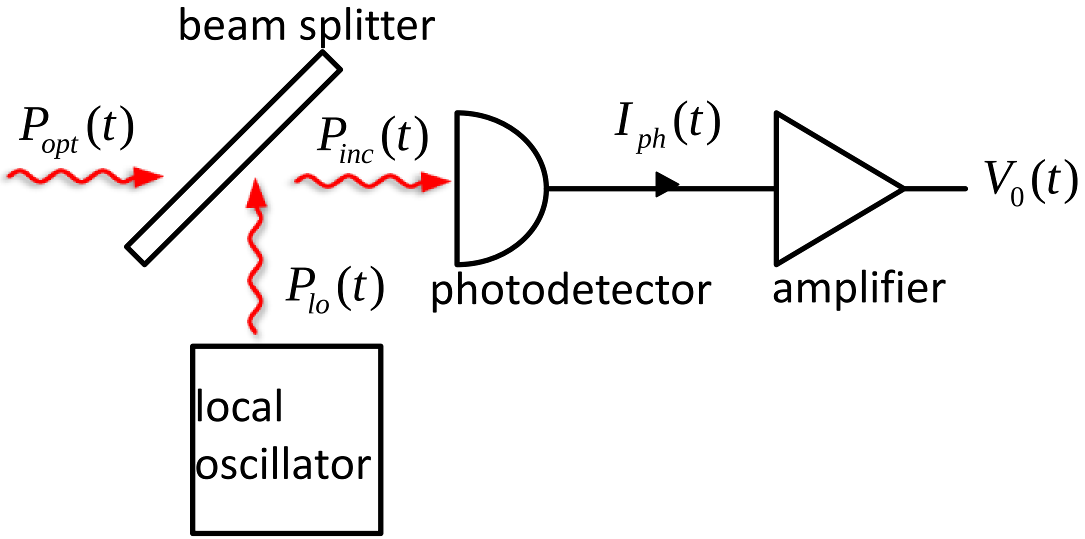
$$= \frac{\eta q}{h\nu}\frac{1}{Z_0}\left|E_{opt}(t) + E_{lo}(t)\right|^2$$

$$\boxed{I_{ph}(t) \cong \frac{\eta q}{h\nu}\left[P_{lo} + 2\sqrt{P_{lo}P_{opt}}\cos\left[(\omega_{opt} - \omega_{lo})t + \Delta\phi\right]\right]}$$

Intermediate frequency (IF)

$$\omega_{if} = \omega_{opt} - \omega_{lo}$$

Coherent detection



$$I_{ph}(t) \cong \frac{\eta q}{h\nu}\left[P_{lo} + 2\sqrt{P_{lo}P_{opt}}\cos\left[\omega_{if}t + \Delta\phi\right]\right]$$

Coherent detection receivers respond to changes in frequency, phase, and intensity

This scheme where the IF is non-zero is known as heterodyne detection.

$$SNR = \frac{I_{ph,rms}^2}{2qI_{ph,lo}B + 4kTBR^{-1}} = \boxed{\frac{2\left(\frac{\eta q}{h\nu}\right)^2 P_{lo}P_{opt}}{2q\frac{\eta q}{h\nu}P_{lo}B + 4kTBR^{-1}}}$$

Coherent vs. direct detection

$$SNR_{dd} = \frac{\left(\frac{\eta q}{h\nu} P_{opt}\right)^2}{2q\frac{\eta q}{h\nu} P_{opt} B + 4kTBR^{-1}} \qquad SNR_{cd} = \frac{2\left(\frac{\eta q}{h\nu}\right)^2 P_{lo} P_{opt}}{2q\frac{\eta q}{h\nu} P_{lo} B + 4kTBR^{-1}}$$

Coherent detection allows you to achieve shot-noise limit even if thermal noise is large. You simply need to increase the local oscillator power (P_{lo}).

In the limit of small thermal noise, heterodyne coherent detection increases the SNR by a factor of two compared with direct direction.

Despite benefits, coherent detection is not always used due to increased cost, power and complexity.